Introduction:

Statistical quality Control abbreviated as S.Q.C is one of the most important applications of the Statistical techniques in industry. These techniques are based on theory of probability and Sampling (நிகழ்தகவு) and are being extensively used in almost all industries.

Eg: Armament (Go), aircraft, automobile, textile, electrical equipment, plastic, rubber, electronic chemicals, petroleum, transportation, medicine and So on.

In fact, it is impossible to think of any industrial field where S.Q.C. is not Used.

Quality - Definition:

The most important Word in the term statistical quality Control is quality. By Quality (Product தரம்) we mean an attribute of the product that determines its fitness for use. The range of these attributes is pretty wide- physical, chemical, aesthetic, etc.

Quality Control here means a level

of the product which in turn, depend on four Ms besides many other factors - materials, Man power, machines and Management.

Need (or) uses of quality Control:

Quality Control is a powerful productivity technique for effective diagnosis of lack of quality in any materials, processes, machines or end Product. It is essential that the end products possess the qualities that the Consumer expects of them for the progress of industry depends on the Successful marketing of Products. Quality Control ensures this by Insisting on quality Specifications all along the line from the arrival of materials, processing to the final delivery of goods.

Quality Control, therefore, covers all the factors and processes of Production which may be broadly classified as follows.

Characteristic of Quality —

i) Quality of materials:

Material of good quality will result in Smooth processing thereby reducing the Waste and increasing the Output. It will also give better finish to the end products.

ii) Quality of Manpower:

Trained and qualified personnal will give increased efficiency due to the better quality production through the application of skill and also reduce Production Cost and Waste

iii) Quality of machines.

Better quality equipment will result in efficient work due to lack or Scarcity of breakdowns and thus reduce the Cost of defectives.

iv) Quality of Management:

A good management is imperative for increase in efficiency, harmony in relations and growth of business and markets

We define quality as fitness for use the two general aspects of quality are:

(i) Quality of Design

(ii) Quality of Conformance

(i) Quality of Design.

All goods and Services are Produced unvarious levels. These Variations in levels of Quality are intentional and they are referred to as quality of Design.

Although all automobiles provide Same transportation, they defer with respect to Size appointments appearance and performance. These differences are the result of intentional design differences. They are due to types to materials used in Construction, tolerances, in manufacturing and other accesories.

(ii) Quality of Conformance.

The Quality of Conformance is how, well the product Conform to the Specifications and tolerance of required by the design. It is influenced by a number of factors such as choice of manufacturing processes the skill of the work force Sophisticated technology etc.

Every product possesses a number of elements that kindly described its fitness for use. These parameters often called quality characteristics.

They may be are Several types.

Physical, length, weight, Voltage, Viscosity.

Sensory, taste, appearance, Colour

Time Orientation, realability, maintainability, Serviceability.

அடிப்படை

Basis of Statistical Quality Control:

Chance Causes: (இயல்பான பருவம்)

Some, "Stable pattern of Variation" or "a Constant cause System" is naturally inherent in any particular Scheme of உற்பத்தி production and மேற்பார்வை inspection. This Pattern result from many minor Causes that behave in a random manner. The Variation due to these causes is beyond the Control of human hand and Cannot be prevented or eliminated under any circumstances one has got to allow for Variation within this Stable pattern. Usually termed as allowable Variation. The range of such Variation is known as 'natural tolerance of the process'.

(குறிப்பிடத்தக்க காரணங்கள்)

Assignable Causes (or) preventable Variation.

The assignable causes of Variation attributed to any production process is due to non-random or the So-called assignable causes and is termed as preventable Variation. The assignable causes may creep in at any stage of the process, right from the arrival of the raw materials to the final delivery of goods. Some of the important factors of a assignable causes of Variation are Sub-standard or defective raw materials.

new techniques or Operations negligence of the Operators, Wrong or improper handling of machines, faulty equipment, unskilled or inexperienced technical staff and so on. These causes can be identified and eliminated are to be discovered in a production process before it goes wrong i.e before the production becomes defective.

Explain difference between chance Causes of Variation & Assignable causes of Variation.

Chance Causes of Variation	Assignable causes of Variation
(i) Consist of many individual causes	Consist of just a few individual causes
(ii) Any one chance cause results in only a small amount of Variation	Any one assignable cause can result in a large amount of Variation.
(iii) Chance Variation cannot economically be eliminated from a process.	The presence of assignable Variation can be detected, and action to eliminated the Causes is Usually economically Justified

iv) Some typical chance causes of Variation are

Some typical assignable causes of Variation are

chance causes of Variation	Assignable causes of Variation
- Slight ~~Varial~~ Vibration of a machine	- Negligence of Operators
- Lack of human perfection in reading instruments and setting Controls	- Defective raw materials
- Voltage fluctuations and Variation in temperatures	- Faulty equipment
	- Improper handling of machines

Statistical quality Control : (S.Q.C)

S.Q.C means planned Collection and effective use of data for Studying causes of Variations in quality either as between processes, procedures, materials, machines, etc., or over Periods of time. This cause - effect analysis is then fed back into the System with a View to Continuous action on the process es of handling, manufacturing, packaging transporting and delivery at end-use.

When do you say process in Control?

A production process is said to be in a state of statistical Control, if it is governed by chance causes alone, in the absence of assignable causes of Variation

Statistical Quality Control: (Definition)

"S.Q.C may be broadly defined as that Industrial management technique by means of which product of Uniform acceptable quality are manufactured. It is mainly Concerned with setting things right rather than discovering and rejecting those made Wrong.

Benefits of Statistical quality Control:

(or) features (or) advantages

The following are some of the benefits that result when a manufacturing process is operating in a State of Statistical Control.

1) An obvious advantage of S.Q.C is the Control, maintenance and improvement in the quality standards.

2) The act of getting a process in

Statistical quality Control involves the identification and elimination of assignable Causes of Variation and possibly the inclusion of good ones.

3) It tells us when to leave a process alone and when to take action to Correct troubles

4) If a process in Control is not good enough, we shall have to make more or less a radical change in the process.

5) A process in Control is predictable - we know what it is going to do and thus we can more safely guarantee the Product.

6) If testing is destructive (eg, testing the breaking strength of chalk; proofing of ammunition, explosives, Crackers, etc) a process in Control gives Confidence in the quality of untested product which is not the case otherwise.

7) It provides better quality assurance at lower inspection Cost.

8) Quality Control finds its applications not only in the sphere of production, but also in other areas like Packaging, Scrap and Spoilage, recoveries, advertising etc. Foreign trade items of developing Countries like India are particularly for every type of quality Control in every Possible area.

9) The very presence of a quality Control Scheme in a plant improves and alerts the personnel

10) S.Q.C reduce waste of time and material to the absolute minimum by giving an early Warning about the Occurance of defects.

Need of S.Q.C

An SQC department is, thus an essential part of a modern plant and its important functions are as follows:

i) Evaluation of quality standards of incoming materials, Products in process and of finished goods.

ii) Judging the Conformity of the process to established standards and taking suitable action when

difference (variation)
deviations are noted.

iii) Evaluation of Optimum quality obtainable under given Conditions

iv) Improvement of quality and Productivity by process Control and experimentation.

5m / 10m / 2m
Uses of S.Q.C.

* Improvement in product Quality and Design.
* Reduction in Operating Cost and Losses.
* Reduction in Scrap (wastage)
* Saving in excess Use of materials
* Removing production bottlenecks
* Reduction in inspection
* Evaluation of scientific tolerances
* Improvement of employee morale
* maintenance of Operating efficiency quality Consciousness
* Greater Customer Satisfaction.

UNIT - II

Process Control:

2m
* The main objective in any Production process is to Control and maintain a Satisfactory quality level of the manufactured product so that it Conforms to Specified quality Standards. In other words, we want to ensure that the proportion of the defective items in the manufactured Product is not too large. This is termed as 'Process Control' and is achieved through the technique of 'Control charts' Pioneered by W.A. Shewhart in 1924.

Product Control:

* Product Control aims at guaranteeing a Certain quality level to the Consumer regardless of what quality level is being maintained by the producer. In other words, it attempts to ensure that the product marketed by Sale department does not Contain a large number of defective (Unsatisfactory) items. Thus, Product Control is Concerned with

Classification of raw material, Semi-finished goods or finished goods into acceptable or rejectable items.

Techniques of SQC Mention techniques of SQC

2m

Process Control (by Control chart Device)		Product Control (by Sampling inspection plans)	
Variables	Attributes	Attributes	Variables
X̄-chart	C-chart		
R-chart	np-chart		
σ-chart (or) S-chart	P-chart		

Control Limits, Specification limits and Tolerance limits Mention type of limits in SQC

5m

1. Control limits

These are limits of Sampling Variation of a statistical measure (eg mean, range, or fraction-defective) Such that if the Production process is under Control, the Values of the measure Calculated from different rational Sub-groups will lie within these limits.

Points falling Outside Control limits indicate that the process is not Operating under a System of chance causes i.e, assignable causes of Variation are present, which must be eliminated. Control limits are used in 'Control charts'.

2. Specification limits:

When an article is proposed to be manufactured the manufacturers have to decide upon the maximum and the minimum allowable dimensions of some quality characteristics so that the product can be gainfully utilised for which it is intended. If the dimensions are beyond these limits, the product is treated as defective and cannot be Used. These maximum and minimum limits of Variation of Individual items, as mentioned in the product Design, are Known as 'Specification limits'.

3. Tolerance limits:

These are limits of Variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie (with a given probability) provided the process is in a state of statistical quality control. For example, we may claim with a probability of 0.99 that at least 90% of the products will have dimensions between some stated limits. These limits are also known as 'statistical tolerance limits'.

Control charts:

i) Development of control charts was made by a young physicist Dr. Walter A. Shewart. ii) Based on the theory of probability and Sampling, Shewart's control charts provide a powerful tool of discovering and correcting the assignable causes of variation outside the 'stable pattern' of chance causes thus enabling us to stabilize and control our processes at desired performances and the process under statistical control. iii) In industry one is faced with two kinds of problems:

* To check whether the process is conforming to standards laid down and
* To improve the level of standard and reduce variability consistent with cost considerations)

[Shewart's control charts provide an answer to both] (iv) Control chart as conceived and devised by Shewart, is a simple pictorial device for detecting unnatural patterns of variations in data resulting from repetitive processes. [Control charts provide a criteria for detecting lack of statistical control] v) [Control charts are simple to construct and easy to interpret] and tells us at a glance whether [the sample point falls within the 3-σ control limits (Discussed below) or not] [Any sample point going outside the 3-σ control limits is an indication of the lack of Statistical Control i.e presence of same assignable causes of Variation which must be traced, identified and eliminated]

vii) A typical Control chart Consists of the following three horizontal lines:

(i) A Central line (C.L) indicating the desired standard.

(ii) Upper Control limit (U.C.L) indicating the upper limit of tolerance.

(iii) Lower Control limit (L.C.L) indicating the lower limit of tolerance.

The Control line as well as the upper and lower limits are established by Computations based on the past records or Current Production records.

Major parts of a Control charts (or Basis of Control Charts:

* A Control chart generally includes the following four major parts.

1. Quality Scale:

Quality Scale is a Vertical Scale. The Scale is marked according to the quality characteristics of each Sample.

2. Plotted Samples:

The qualities of individual items of a Sample are not shown on a Control chart. Only the quality of the entire sample represented by a Single Value is plotted. The Single Value is plotted on the chart is in the form of a dot (Sometimes a Small Circle or Cross)

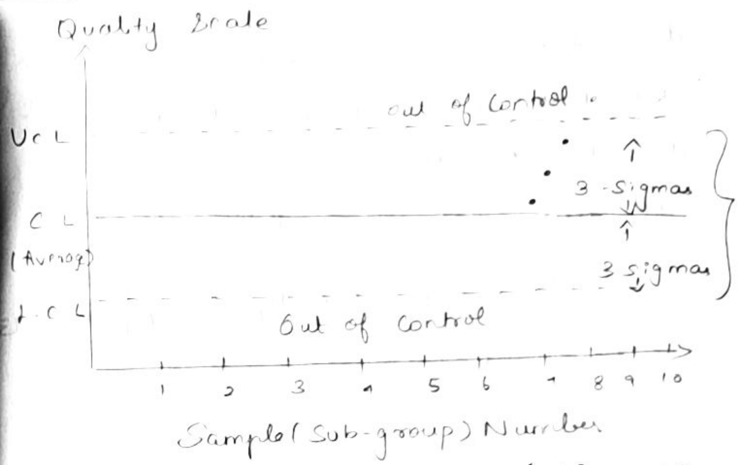
3. Sample (or Sub-group) Numbers:

The Samples are plotted on a Control chart are numbered individually and Consecutively on a horizontal line. The line is Usually placed at the bottom of the chart. The Samples are also reffered to as Sub-groups in Statistical quality Control. Generally 25 Sub-groups are used in Constructing the Control Chart.

4. The horizontal lines:

The Central line represent the average quality of the samples plotted on the chart. The line above the Central line shows the Upper Control limit (UCL) which obtained by adding 3 Sigma's to the average. i.e Mean + 3(S.D). The line below the central line is called the lower Control limit (L.C.L) which is obtained by Subtracting 3 Sigma's from the average i.e Mean − 3 (S.D). The Upper and lower Control limits are Usually drawn as dotted lines, and the Central line is plotted as a (dark) line.

Outline of a Control chart



If t is the statistic then these Values depend on the Sampling distribution of t and are given by

$Z = \frac{X - E(X)}{\sqrt{V(X)}} = \frac{t - E(t)}{\sqrt{V(t)}} = \frac{t - \mu_t}{\sqrt{\sigma_t^2}} = \frac{t - \mu_t}{\sigma_t}$

$U.C.L = E(t) + 3 \cdot S.E(t) \Rightarrow \mu_t + 3\sigma_t$

$L.C.L = E(t) - 3 \cdot S.E(t) \Rightarrow \mu_t - 3\sigma_t$

$C.L = E(t) \Rightarrow \mu_t$

3-σ Control charts:

3-σ limits were proposed by Dr. Shewhart for his Control charts from Various Considerations, the main being probabilistic Considerations.

Consider the statistic $t = t(x_1, x_2 \ldots x_n)$ a function of the Sample Observations $x_1, x_2 \ldots x_n$)

Let, $E(t) = \mu_t$ and $Var(t) = \sigma_t^2$

If the statistic t is normally

distributed, then from the fundamental area property of the normal distribution.

$P(-3 < z < 3) = 0.9973$

We have

$P(-3 < \frac{t-\mu t}{\sigma t} < 3) = 0.9973$

$$P[\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t] = 0.9973$$

$P(-3\sigma_t < t - \mu t < 3\sigma t)$.

$$\Rightarrow P[|t - \mu_t| < 3\sigma_t] = 0.9973$$

$P[\mu_t - 3t < t < \mu_t + 3t]$.

i.e, $P[|t - \mu_t| > 3\sigma_t] = 0.0027$

In otherwords, the probability that the random value of t goes outside the 3-σ limits, $\mu_t \pm 3\sigma_t$ is 0.0027, which is very small. Hence if t is normally distributed, the limits of variation between $\mu_t + 3\sigma_t$ and $\mu_t - 3\sigma_t$ which are termed respectively the Upper Control limit (U.C.L) and Lower Control limit (L.C.L)

If the i^{th} sample, the Hence if t is normally Distributed the limits lies between the upper and lower control limits, there is nothing to worry as in such a case variation between samples is attributed to chance i.e In this case the process is in statistical control.

(ii) It is only when any observed the i^{th} sample of t_i falls outside the control limits, it is considered to be a danger signal indicating that some assignable cause has crept in which must be identified and eliminated.

Tools for SQC

$\sigma_t \Rightarrow$ Sample Variance

The following four, seperate but related techniques are the most important statistical tools for data analysis in quality control of the manufactured products

1. Shewhart's Control chart for Variable

i.e for a characteristic which can be measured quantitatively. Many quality characteristic of a product are measurable and can be expressed in specific units of measurements such as diameter of a screw, tensile strength of steel pipe, specific resistance of a wire, life of an electric bulb, etc Such Variables are of Continuous type and are regarded to follow normal probability law. For quality Control of Such data, two types of Control charts are used and technically these charts are known as:

(a) charts for $\bar{x}$ (mean) and R (Range) and,

(b) charts for $\bar{x}$ (mean) and σ (standard deviation).

2. Shewhart's Control chart for <u>fraction Defective</u> or p-chart:

This chart is used if we are dealing with attributes in which case the quality characteristics of the product are not <u>amenable</u> (suitable) to measurement but can be identified by their absence or presence from the product or by classifying the product as defective or non-defective.

3. Shewhart's Control charts for the '<u>Number of Defects</u>' per Unit or C-chart.

This is Usually used with advantage when the characteristic representing the quality of a product is a <u>discrete Variable</u>.

Eg: (i) The number of defective rivets in an aircraft wing, and
(ii) The number of surface defects Observed in a roll of Coated paper or a sheet of photographic film.

4. The portion of the sampling theory which deals with the quantity protection given by any specified sampling acceptance procedure.

Control charts for Variables:

These charts may be applied to any quality characteristic that is measurable. In order to Control a measurable characteristic we have to exercise Control on the measure of location as well as the measure of <u>dispersion</u>.

Usually $\bar{x}$ and R charts are employed to Control the mean (location) and standard deviation (dispersion) respectively of the characteristic

$\bar{x}$ and R charts:

No production process is perfect enough to produce all the items exactly alike. Some amount of variation, in the produced items, is inherent in any production scheme. This variation is the totality of numerous characteristic of the production process viz., raw material, machine setting and handling, operators, etc as pointed out earlier, this variation is the result of

(i) chance causes

(ii) assignable causes

The control limits for $\bar{x}$ and R charts are so placed that they reveal the presence (or) absence of assignable causes of variation in the

(a) average - mostly related to machine - setting, and

(b) range - mostly related to negligence on the part of the operator.

Steps for $\bar{x}$ and R charts

1. Measurement:

The first work for control chart starts with measurements. Any method of measurement has its own inherent variability. Errors in the measurement can enter into the data by

(i) the use of faulty instruments,

(ii) lack of clear cut definitions of quality characteristics and the method of taking measurements.

(iii) lack of experience in handling (or) use of the instrument.

2. Selection of subgroups or sample or rational forming the sub-groups:

In order to make the control chart analysis effective, it is essential to pay due regard to the rational selection of samples or sub-groups. The choice of the sample size n and the frequency of sampling, i.e. the time between the selection of two groups, depend upon the process.

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Usually n is taken to be 4 or 5 the frequency of sampling depends on the state of the control exercised. Normally 25 samples of size 4 each or 20 samples of size 5 each under control will give good estimate of the process average and dispersion.

3. Calculation of $\bar{x}$ and R for each sub-group

Let x_{ij}, $j = 1, 2, \ldots n$, $i = 1, 2, \ldots k$ be the measurements on the i^{th} sample. The mean $\bar{x}_i$ the Range R_i and the standard deviation s_i respectively as follows

$$\bar{x}_i = \frac{1}{n} \sum_j x_{ij}$$

$$R_i = \max_j x_{ij} - \min x_{ij}$$

$$s_i^2 = \frac{1}{n} \sum_j (x_{ij} - \bar{x}_i)^2$$

Next we find $\bar{\bar{x}}$, $\bar{R}$, $\bar{s}$ the averages of sample means, sample ranges and sample standard deviation, respectively as follows.

$$\bar{\bar{x}} = \frac{1}{k} \sum_i \bar{x}_i,$$

$$\bar{R} = \frac{1}{k} \sum_i R_i,$$

$$\bar{s} = \frac{1}{k} \sum_i s_i$$

4. Setting of control limits:

It is well known that if σ is the process standard deviation (standard deviation of the universe from which samples are taken), then the standard error of sample mean is $\sigma/\sqrt{n}$, where n is the sample size,

$$S.E(\bar{x}_i) = \sigma/\sqrt{n} \quad (i = 1, 2 \ldots k)$$

Also from the sampling distribution of range, we know that

$$E(R) = d_2 . \sigma$$

where d_2 is a constant depending on n the sample size. Thus an estimate of σ can be obtained from $\bar{R}$ by the relation

$$\bar{R} = d_2 . \sigma \Rightarrow \hat{\sigma} = \bar{R}/d_2$$

Also $\bar{\bar{x}}$ gives an unbiased estimate of the population mean μ, since

$$E(\bar{\bar{x}}) = \frac{1}{k} \sum_{i=1}^{k} E(\bar{x}_i) = \frac{1}{k} \sum_{i=1}^{k} \mu = \mu$$

Control limits for $\bar{x}$-charts

Case 1:

When standards are given, i.e. both μ and σ are known the 3-σ Control limits for $\bar{x}$ charts is given by

$$E(\bar{x}) \pm 3 S.E(\bar{x}) = \mu \pm 3\sigma/\sqrt{n} = \mu \pm A\sigma,$$

If μ' and σ' are known or specified values of μ and σ respectively then

$$UCL_{\bar{x}} = \mu' + A\sigma' \text{ and } LCL_{\bar{x}} = \mu' - A\sigma'$$

where $(A = 3/\sqrt{n})$ is a Constant depending on n and its values are tabulated for different values of n from 2 to 25 in Table VIII in the Appendix

Case 2:

μ and σ are Unknown.

(i) When standards not given. If both

(ii) If μ and σ are unknown then using their estimates $\bar{\bar{x}}$ and $\hat{\sigma}$ given in Eqns $\bar{\bar{x}} = \frac{1}{k}\sum_i \bar{x}_i$, $\bar{R} = \frac{1}{k}\sum_i R_i$, $\bar{s} = \frac{1}{k}\sum_i s_i$ and $\bar{R} = d_2 . \sigma$

$\Rightarrow \hat{\sigma} = \bar{R}/d_2$ respectively

We get the 3-σ control limits the $\bar{x}$-chart as

$$\bar{\bar{x}} \pm 3\frac{\bar{R}}{d_2}\cdot\frac{1}{\sqrt{n}} = \bar{\bar{x}} \pm \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R} = \bar{\bar{x}} \pm A_2 \bar{R}$$

$$\left(A_2 = \frac{3}{d_2\sqrt{n}}\right)$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \text{ and}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

[Values have been Computed and tabulated for different values of n from 2 to 25]

→ Since d_2 is a Constant depending on n, $A_2 = 3/d_2\sqrt{n}$) also depends only on n' i.e. its ↑

Control limits for R-charts:

(i) R-chart is Constructed for Controlling the Variation in the dispersion of the product] The procedure of Constructing R-chart is similar to that for the $\bar{x}$-chart and (ii) R-Chart involves the following steps

1: Compute the range $R_i = \max_j x_{ij} - \min x_{ij}$.

2. Compute the mean of the sample ranges

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i = \frac{1}{k}(R_1 + R_2 + \cdots + R_k) \quad (i = 1, 2, \ldots k),$$

3. Computation of Control limits

The 3-σ Control limits for R-chart is given by $E(R) \pm 3\sigma_R$.

$E(R)$ is estimated by $\bar{R}$ and σ_R is estimated from the relation

$$\sigma_R = d_3 \hat{\sigma} = d_3 \frac{\bar{R}}{d_2}$$

where d_2 and d_3 are Constants depending on n.

$$UCL_R = E(R) + 3\sigma_R = \bar{R} + \frac{3d_3}{d_2}\bar{R}$$

$$UCL_R = \left(1 + \frac{3d_3}{d_2}\right)\bar{R} = D_4\bar{R}$$

Similarly,

$$LCL_R = \left(1 - \frac{3d_3}{d_2}\right)\bar{R} = D_3 = \bar{R}$$

The Values of D_4 and D_3 depend on n and have been Computed and tabulated for different Values of n from 2 to 25 in table given at the end of the chapter

However if σ is known, then

$$UCL_R = E(R) + 3\sigma_R = d_2\sigma + 3d_3\sigma = (d_2 + 3d_3)\sigma = D_2\sigma$$

$$LCL_R = E(R) - 3\sigma_R = d_2\sigma - 3d_3\sigma = (d_2 - 3d_3)\sigma = D_1\sigma$$

In each Case (σ known or Unknown) the Central line is given by

$$CL_R = E(R) = \bar{R}$$

Since range can never be negative, LCL must be greater than or equal to 0. [In Case it Comes out to be negative, it is taken as zero.]

4. Construction of Control charts for $\bar{x}$ and R.

i.e plotting of Central line and the Control limits

Control charts are plotted on a rectangular Co-ordinate axis - vertical Scale representing the statistical measures $\bar{x}$ and R, and horizontal Scale representing the sample number.

For $\bar{x}$-chart the central line is drawn as a solid horizontal line at $\bar{\bar{x}}$ and $UCL_{\bar{x}}$ and $LCL_{\bar{x}}$ are drawn at the computed values as dotted horizontal lines.

For $\bar{R}$-chart, the central line is drawn as a solid horizontal line at $\bar{R}$ and UCL_R is drawn at the computed value as a dotted horizontal line. If the sample size is seven or more $(n \geq 7)$, LCL_R is drawn as dotted horizontal line at the computed value, otherwise $(n<7)$ LCL_R is taken as zero.

Modified Control limits for future use:

If all the points in both the charts remain within trial control limits, then these limits are accepted as final, and used for maintaining control charts for subsequent production. If, however, some of the points go outside the limits in one of the charts then it is concluded that these samples were produced when the process was not in control and these samples are rejected as unusable. Then a second set of trial limits is constructed, using only the remaining samples and using these fresh control limits, new charts are constructed and the remaining samples are plotted on the new charts. If all the sample points now remain within the new control limits, they are accepted as final otherwise the same procedure as described above is followed to get a third set of trial control limits. The control limits are accepted as final only when all the sample points on which they are based remain within these limits.

Criterion for Detecting lack of Control in $\bar{x}$ and R-charts:

As pointed out earlier, the main object of the control chart is to indicate when a process is not in Control. The Criteria for detecting lack of Control are, therefore, of fundamental and Crucial importance. The pattern of the Sample points in a Control chart is the key to the proper interpretation of the Working of the process. The following Situations depict lack of Control.

1. A point Outside the Control limits:

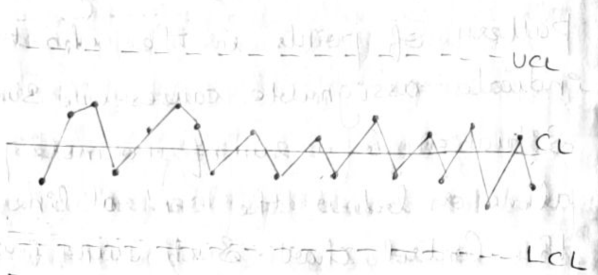
The probabilistic Considerations provide a basis for hunting for lack of Control in Such a Situation. A point going Outside Control limits is a clear indication of the presence of assignable Causes of Variation which must be searched and Corrected. A point Outside the Control limits may result from an increased dispersion or change in level. Lack of Uniformity may be due to the Variation in the quality of raw materials, deficiency in the skill of the Operators, loss of alignment among machines, change of working Conditions etc It may be indicated by a point (or points) above the upper Control limit for ranges. It may also result in points Outside the Control limits for means.

2. A run of Seven or more points:

Although all the Sample points are within Control limits, Usually the pattern of points in the chart indicates assignable causes. One Such Situation is a run of 7 or more points above or below the central line in the Control chart. Such runs indicate shift in the process level. on R-chart a run of points above the central line is indicative of Increase in process Spread and therefore represents an undesirable Situation, while a run below the Central line indicates an improvement.

Control procedure for $\bar{x}$ and R-charts

4. The Sample points on $\bar{x}$ and R-charts too close to the Central line, exhibit another form of assignable cause. This situation represents Systematic differences within Samples or Sub-groups and results from improper Selection of Samples and biases in measurements.



5. Presence of trends:

The trends exhibited by Sample points on the Control chart are also an indication of assignable cause. Trend pattern, a phenomenon Usually observed in engineering industry, indicates the gradual shift in the process level. Trend may be Upward or downward. Tool wear and the need for resetting machines often accounts for such a shift, and it is essential to determine when machine resetting becomes desirable bearing in mind that too frequent adjustment are a serious set back to production.

Presence of trends in Control chart

6. Presence of Cycles:

In Some cases the Cyclic pattern of points in the Control chart indicates the presence of assignable Causes of Variation. Such patterns are due to material or /and any mechanical reasons

Presence of Cycles in Control charts

Control chart for Standard Deviation (or σ-chart)

(1) Since standard deviation is an ideal measure of dispersion, a Combination of Control chart for mean ($\bar{x}$) and standard Deviation (σ) or s, known as $\bar{x}$ and s-charts (or $\bar{x}$ and σ-charts) is (2) theoretically more appropriate Combination of $\bar{x}$ and R-charts for Controlling process average and process Variability.

In a random sample of size n from normal population with standard deviation σ,

(3) We have,

$$E(s^2) = \frac{n-1}{n}\sigma^2$$

and $E(s) = c_2 \cdot \sigma$

where,

$$c_2 = \sqrt{\frac{2}{n}} \cdot \frac{[(n-2)/2]!}{[(n-3)/2]!}$$

Where, c_2 have been tabulated for different Values of n from 2 to 25 in the table at the end of the chapter

Hence, in sampling from normal population

(4) We have,

$$Var(s) = E(s^2) - \{E(s)\}^2 = \left(\frac{n-1}{n} - c_2^2\right)\sigma^2$$

$$\Rightarrow S \cdot E(s) = c_3 \cdot \sigma$$

Where,

$$c_3 = \sqrt{\frac{n-1}{n} - c_2^2}$$

$$UCL_s = E(s) + 3\, S.E(s) = (c_2 + 3c_3)\sigma = B_2 \cdot \sigma$$

$$LCL_s = E(s) - 3 \cdot S \cdot E(s) = (c_2 - 3c_3)\sigma = B_1 \sigma$$

Central line $= CL_s = c_2 \cdot \sigma$

Where B_1 and B_2 have been tabulated for different Values of n

(5) If the Value of σ is not specified or not known, then we use its estimate, based on $\bar{s}$ is defined is given by,

$$\hat{\sigma} = \bar{s}/c_2$$

$$UCL_s = E(s) + 3\; S \cdot E(s) = \bar{s} + 3\frac{c_3}{c_2} \cdot \bar{s}$$

$$= \left(1 + \frac{3c_3}{c_2}\right)\bar{s} = B_4 \cdot \bar{s}$$

Similarly, we shall get

$$LCL_S = \left(1 - \frac{3C_3}{C_2}\right)\bar{s} = B_3 \cdot \bar{s}$$

and $CL_S = \bar{s}$

where B_3 and B_4 have been tabulated for different values of n. Since s can never be negative, if LCL given by $CL_S = \bar{s}$ to be negative. In this case for n from 2 to 5, it is taken to be zero.

Difference b/w S & R-chart

(8) S-chart vs R-chart:

5m (or) 2m.

In small samples the standard deviation S and range R are likely to fluctuate together, i.e if S is small (large) R is likely to be small (large). However for large samples a single extreme observation will have a significantly large effect on range while its effect on standard deviation will be comparatively much less. Hence for analysing or controlling variability if we use small samples, the range may be used as a substitute for standard deviation with little loss in efficiency. Since range is almost as efficient as standard deviation in small samples it is usually preferred to standard deviation in quality control analysis because of its ease of calculations.